Online Supplement: Age, Period, and Cohort Analysis with Bounding and Interactions

S1 Additional Data Details

The universe of VRSs is all US citizens aged [1](#page-1-0)8 and above eligible for the basic CPS.¹ Among a total of 1,136,990 respondents in the universe, 99,163 respondents (or 8.7 percent) are dropped due to non-response to either the VRS itself or the question on voting. An additional 55,969 respondents aged 78 or above at the time of the survey were dropped because, for some years, CPS has top-coded age for all respondents aged 80 or above; therefore, precise and consistent four-year age and cohort intervals could not be constructed for these individuals. These exclusions result in a final analytical sample of 981,858 respondents (see Table S1 for age-by-period-group-specific n in the analytical sample).

As the outcome variable, I use responses to the question asked to all respondents: "In any election, some people are not able to vote because they are sick or busy or have some other reason, and others do not want to vote. Did (you/name) vote in the election held on [election date]? Respondents could answer from the list: 1) Yes; 2) No; 3) Refuse; 4) Don't know. Respondents are coded 1 if they answered "yes" and 0 if "no." "Refuse" and "Don't know" are considered missing and hence are dropped. Given the period between presidential elections, I group all age and cohort groups into four-year intervals so that all age, period, and cohort groups are of equal width. The cohort variable is constructed by subtracting age from the survey year (i.e., cohort = year - age).

A potential limitation of CPS-VRSs is that non-response rates have increased over time. Most non-response occurs because respondents opt not to participate in VRSs altogether. This behavior has become increasingly common over time, as evident in Figure [S1a.](#page-6-0) $^{\rm 2}$ $^{\rm 2}$ $^{\rm 2}$ I adjust for the non-response via inverse probability weighting. First, I estimate election-specific logistic regressions predicting the probability of non-response using various background variables gathered in the basic CPS and outlined in Section S4. Then, the respondents of VRSs are weighted by the product of the base sampling weight *VOSUPPWT* and the (multiplicative) inverse of their response probabilities (i.e., $1/(1 - \hat{p}(\text{non-response})).$ This weighting ensures that the distribution of the observed characteristics in the analytical sample aligns with the basic CPS. I apply this weight to all results presented in the article; however, as shown in Figure [S1b.](#page-6-0) the impact of non-response is minor on the overall findings.[3](#page-1-2)

¹However, among the 1,160,569 respondents who meet these criteria in the selected years for analysis, an additional 23,579 respondents are categorized as "Not in Universe" for reasons that are not explicitly stated in the CPS-VRSs.

 2 In its official reports, the Census Bureau has typically coded these non-responses uniformly as non-voters, with the justification that such practice leads to more accurate estimates of the actual turnout rate by offsetting overreporting of turnout by the respondents. However, with the gradual rise in non-response, this justification is increasingly being questioned with such practice, leading to unreliable estimates of the overall turnout rate [\(Hur and Achen](#page-12-0) [2013\)](#page-12-0).

³Another data limitation is that all voting participation is self-reported. Overreporting bias can be sizeable in selfreports of turnout, even in CPS, which suffers less from the bias compared to other academic surveys [\(DeBell, Krosnick,](#page-12-1) [Gera, Yeager, and McDonald](#page-12-1) [2020;](#page-12-1) [Hur and Achen](#page-12-0) [2013\)](#page-12-0). Without validated vote records, this limitation cannot be fully addressed. Therefore, all findings should be interpreted with this caveat in mind.

S2 Details of the Bounding Analysis

[Fosse and Winship](#page-12-2) [\(2019\)](#page-12-2) show that imposing an initial bound on the linear age slope $\alpha_L\in[\alpha_L^{min},\alpha_L^{max}]$ yields the following partial identification of the other linear slopes:

$$
\alpha_L^{min} \le \alpha_L \le \alpha_L^{max} \tag{1a}
$$

$$
(\alpha_L + \beta_L) - \alpha_L^{max} \le \beta_L \le (\alpha_L + \beta_L) - \alpha_L^{min}
$$
\n(1b)

$$
(\gamma_L - \alpha_L) + \alpha_L^{min} \le \gamma_L \le (\gamma_L - \alpha_L) + \alpha_L^{max}
$$
\n(1c)

Similarly, bounds on the linear cohort slope $\gamma_L \in [\gamma_L^{min},\gamma_L^{max}]$ yields

$$
(\alpha_L - \gamma_L) + \gamma_L^{min} \le \alpha_L \le (\alpha_L - \gamma_L) + \gamma_L^{max}
$$
 (2a)

$$
(\beta_L + \gamma_L) - \gamma_L^{max} \le \beta_L \le (\beta_L + \gamma_L) - \gamma_L^{min}
$$
 (2b)

$$
\gamma_L^{min} \le \gamma_L \le \gamma_L^{max} \tag{2c}
$$

In the data, $\big((\alpha_L+\beta_L),(\beta_L+\gamma_L),(\alpha_L-\gamma_L),(\gamma_L-\alpha_L)\big)$ is estimated to be (0.77, 0.19, 0.59, -0.59).

As shown in the main text, Assumptions A1 and A2 yield $\alpha_L \in [0.40, 0.82]$. Plugging in the lower and upper bounds into Equations [\(1a\)](#page-2-0)-[\(1c\)](#page-2-1) yields

$$
0.40 \le \alpha_L \le 0.82, \ -0.04 \le \beta_L \le 0.37, \ -0.18 \le \gamma_L \le 0.23
$$

Assumption C1 asserts that the linear cohort slope is non-negative and therefore implies $\gamma_L^{min}=0$. Plugging in this lower bound to Equations [\(2a\)](#page-2-2)-[\(2c\)](#page-2-3) yields

$$
0.59 \le \alpha_L < \infty, \ -\infty < \beta_L < 0.19, \ 0 \le \gamma_L < \infty
$$

Assumption C2 stipulates that the (weighted) average cohort effect parameters of respondents born between 1911 and 1926 are greater than or equal to those born between 1947 and 1966. To illustrate how this assumption bounds the linear cohort effect, let $s \in S = \{1911\text{-}1914,\dots\}$ 1923-1926 index the cohort groups between 1911 and 1926 and $t \in T = \{1947-1950, \ldots, 1963-1966\}$ index the cohort groups from 1947 to 1966.

The assumption implies

$$
\sum_{s \in S} w_s \gamma_s \ge \sum_{t \in T} w_t \gamma_t
$$

where $w_s = n_s / \sum$ s∈S n_s (n_s denote the number of respondents of cohort $s; w_t$ defined similarly). 4 4

⁴Precisely, n_s is the number of respondents in cohort s after applying the sampling weight described in section S1.

The linear cohort slope bound γ_L^{max} implied by Assumption C2 is informed by:

$$
\sum_{s} w_{s} \gamma_{s} \geq \sum_{t} w_{t} \gamma_{t} \Rightarrow \sum_{s} w_{s} (\gamma_{L} C_{s}^{L} + \tilde{\gamma}_{s}) - \sum_{t} w_{t} (\gamma_{L} C_{t}^{L} + \tilde{\gamma}_{t}) \geq 0
$$

$$
\Rightarrow \gamma_{L} \left(\sum_{s} w_{s} C_{s}^{L} - \sum_{t} w_{t} C_{t}^{L} \right) + \sum_{s} w_{s} \tilde{\gamma}_{s} - \sum_{t} w_{t} \tilde{\gamma}_{t} \geq 0
$$

$$
\Rightarrow \gamma_{L} \leq \frac{\sum_{t} w_{t} \tilde{\gamma}_{t} - \sum_{s} w_{s} \tilde{\gamma}_{s}}{\sum_{s} w_{s} C_{s}^{L} - \sum_{t} w_{t} C_{t}^{L}}
$$

since $\sum_s w_s C_s^L - \sum_t w_t C_t^L > 0.$ The derivation yields $\gamma_L^{max} = 0.03.$

Plugging this into the bounding formula informs

$$
-\infty < \alpha_L \le 0.61, \ \ 0.16 \le \beta_L < -\infty, \ \ -\infty < \gamma_L \le 0.03
$$

The bounds implied jointly by the assumptions can be derived by taking the intersection of the lower and upper bounds.

S3 Some Considerations in Causal Interpretations of APC Parameters

In APC analysis, the terms "effects" are commonly used to describe variations in age, period, and cohort. Yet, the literature consistently acknowledges that APC variables mainly serve as "proxies" or "surrogate indexes" of more concrete underlying causal states [\(Heckman and Robb](#page-12-3) [1985;](#page-12-3) [Ryder](#page-12-4) [1965\)](#page-12-4). This brief discussion highlights some challenges to assigning causal interpretations to the APC variables, particularly when viewed from the modern notions of counterfactual causality.

Central to the challenge is the functional dependence of the APC variables. When considered jointly, the linear dependency of the APC variables provides a tenuous foundation for defining causal effects. Counterfactual models of causality define causal effects as the differences in counterfactual outcomes under hypothetical interventions (see for review, [Morgan and Winship](#page-12-5) [2015\)](#page-12-5). This definition underscores the need for well-defined counterfactual states where all individuals of the target population could, at least in theory, be exposed. However, formulating such causal definitions in APC analysis is a formidable challenge because the functional dependence of the APC variables complicates envisioning interventions on one variable without altering the others. This intrinsic dependence contradicts the "modularity" assumption of Pearl's Structural Causal Model, a principle suggesting that a change in one variable (due to intervention) should not affect the existing causal relationships of other variables within the system [\(Pearl](#page-12-6) [2009:](#page-12-6)24).

The APC-I model could provide a firmer foundation for causal definitions, avoiding the linear dependency by its conceptual refinement. However, additional complications remain. Notably, the APC variables are inherently "compound treatments," encapsulating a multitude of specific causal states (Hernán and VanderWeele [2011\)](#page-12-7). The consistency assumption of the potential outcomes framework for causal inference mandates that for any outcome Y , each hypothetical intervention setting a treatment X to a certain level x should invariably produce a singular counterfactual outcome $Y(x)$ for each individual in a population (see [VanderWeele](#page-12-8) [2009\)](#page-12-8). Ambiguities arise when multiple conceivable ways exist to set $X = x$, each potentially yielding different potential outcomes. In practice, this condition requires that the causal states under consideration must be well-specified, removing any ambiguities regarding its inherent features that might result in varied potential outcomes [\(VanderWeele](#page-12-9) [2018\)](#page-12-9)

The practical application of the consistency condition in APC analysis is debatable, even by the pragmatic standards of social sciences. Consider the 2020 period effect on voter turnout, characterized by unique events like the COVID-19 pandemic, George Floyd's death, and the polarizing presence of Donald Trump. Every one of these events potentially resonates differently with individuals, influencing their voter turnout in unique ways. Envisioning a hypothetical intervention setting an individual's period profile to 2020 is thus challenging due to the multitude of events defining that year. Each interpretation of this "2020 exposure" will likely produce different potential outcomes, thereby not sufficiently aligning with the consistency assumption.

In such contexts, the APC estimand could be articulated as a weighted average of the effects of different exposure versions under the assumption of no unobserved confounding [\(VanderWeele](#page-12-10) [and Hernan](#page-12-10) [2013\)](#page-12-10). These weights would be defined by the probability of each version naturally occurring in subpopulations that have been exposed to the event or condition in question (see also [VanderWeele](#page-12-9) [2018\)](#page-12-9). However, even with this approach, the underlying distribution of the versions of exposure must be carefully considered or even explicitly articulated to achieve more precise interpretations.

S4 Non-response in CPS-VRSs and Comparison of the Actual Turnout Rate with Self-Reported Turnout Rates

To assess the impact of non-response and self-reporting of voting participation in the CPS-VRSs, I present Figure S1. Panel (a) displays the proportion of non-response in CPS-VRSs from 1976 to 2020. Panel (b) presents the 1) estimates of the actual voting-eligible population (VEP) turnout rates from [McDonald](#page-12-11) [\(2021\)](#page-12-11) (plotted in green), 2) CPS self-reported turnout rates using base survey weights (in blue), and 3) CPS self-reported turnout rates using IPTW that adjusts non-respondents in the VRSs (in red). To construct the IPTW weights, I used background variables consistently collected in the basic CPS from 1976 to 2020, including age, race/ethnicity, gender, state, citizenship status (i.e., born as a citizen or naturalized), marital status, number of children in household, metropolitan status, and education, occupation, and family income.

I find few notable patterns. First, panel (a) confirms that the non-response rate has gradually increased over the observational interval. While the non-response rate was as low as 5.4 percent in 1976, the rate rose to 15.7 percent in 2020. Second, by comparing the red dashed line to the solid blue line in panel (b), it is evident that the IPTW weighting to adjust for non-response has a minor impact on the overall turnout rate. In general, such adjustment brings down the overall turnout rate of each election by at most 1.6 percentage points. Third, I observe that the respondents over-report their actual voting status over the entire observational period; however, the magnitude of over-reporting has been relatively stable over time. The election-specific over-reporting has fluctuated between 7.9 percentage points (in 1976) to 11.6 percentage points (in 2008). This result suggests that while the over-reporting may lead to imprecise estimates of the level turnout rates, such bias may not be as

(b) Turnout Rates Comparisons

Figure S1: Trends in Non-response in CPS-VRSs and Comparison of Turnout Estimates

Note: Panel (a) plots the proportion of non-response in CPS-VRSs from 1976 to 2020. Panel (b) plots the election-specific turnout rates based on the 1) base sampling weight (in blue), the IPTW weights (in red), and 3) the actual VEP turnout rate obtained from [McDonald](#page-12-11) [\(2021\)](#page-12-11).

consequential in comparing the relative turnout levels across the APC groups.

S5 Age-by-Period Observations in the Analytic Sample

	Election Year											
	1976	1980	1984	1988	1992	1996	2000	2004	2008	2012	2016	2020
Age												
$18 - 21$	8,564	11,014	8,481	6,786	6,197	5,046	4,860	5,501	5,055	4,917	4,474	3,893
$22 - 25$	8,269	10,980	9,224	7,519	6,888	5,084	4,529	5,386	5,007	5,107	4,948	3,651
$26 - 29$	7,936	10,942	9,565	8,446	7,432	5,619	4,626	5,116	5,103	5,169	4,984	4,123
$30 - 33$	6,650	10,455	9,222	8,793	8,638	6,333	5,174	5,562	4,819	5,406	5,190	4,412
34-37	5,937	8,353	8,590	8,326	8,300	7,082	5,818	6,195	5,135	4,963	5,304	4,391
38-41	5,434	7,175	6,867	7,859	8,160	7,083	6,567	6,867	5,698	5,223	4,836	4,319
$42 - 45$	4,949	6,323	6,124	6,314	7,535	6,709	6,601	7,419	6,051	5,720	4,880	4,093
46-49	5,387	6,118	5,433	5,387	6,202	6,160	6,114	7,433	6,584	5,954	5,158	4,120
$50 - 53$	5,541	6,732	5,096	4,900	5,320	4,848	5,638	6,864	6,589	6,599	5,736	4,392
54-57	5,166	6,681	5,473	4,439	4,650	4,103	4,464	6,475	6,323	6,422	6,144	4,620
58-61	4,587	6,283	5,314	4,746	4,397	3,632	3,702	5,099	5,720	5,925	6,024	5,130
62-65	4,206	5,658	5,100	4,682	4,523	3,293	3,385	4,209	4,322	5,507	5,700	5,097
66-69	3,577	4,993	4,334	4,424	4,311	3,374	2,921	3,523	3,532	4,237	5,091	4,637
$70 - 73$	2,907	4,120	3,739	3,755	3,813	3,137	2,925	3,072	2,913	3,373	3,898	4,192
74-77	2,159	3,076	2,944	2,980	2,864	2,605	2,512	2,815	2,651	2,693	2,938	3,022

Table S1: Unweighted Number of Respondents by Election Year and Age Groups

S6 Additional Figures

Note: The blue shaded areas represent the bounding analysis identification intervals. The red circles represent the APC-I model's point estimates. The dashed horizontal lines represent the intercepts. Survey weights augmented by adjustments for non-response (N=981,858).

Figure S2: APC Bounding Analysis Based on Logistic Regression Models (for Comparison with Figure 5)

Note: These estimates are derived by fitting a linear orthogonal polynomial contrast to the age-period interaction terms corresponding to each birth cohort. The estimates, therefore, summarize the evolving trends of turnout rates within each cohort throughout their live course. The circles represent point estimates, while the vertical bars indicate their corresponding 95-percent point intervals. For the two boundary cohorts (i.e., 1889-1992 and 2001-2002 cohorts), the estimates are not defined as they appear in only one election within the data. Survey weights augmented by adjustments for non-response (N=981,858).

S7 The Full List of the Estimates

Table S2: Nonlinear APC Components (for Comparison with Figure 4)

Note: The estimates correspond to $\tilde{\alpha}_a, \tilde{\beta}_p,$ and $\tilde{\gamma}_c$ of Equation (2) in the main text (N=981,858).

Table S3: APC-I Model and Bounding Analysis Estimates for the Total APC Effect Parameters (for Comparison with Figure 5)

Note: The bounding analysis estimates correspond to $\alpha_a, \beta_p,$ and γ_c of Equation (1) in the main text (N=981,858). The APC-I estimates correspond to or derived from α_a^* , β_p^* , and η_{ap}^* of Equation (4) in the main text (N=981,858). Standard errors in parentheses.

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